Abstract: The article presents Value at Risk (VaR) the measurement method of market risk, one of the most important risk measurement methods in banking sector. Beginning with the VaR definitions existing in the specialty literature, the most common methods in calculating VaR are detailed, such as: Parametric VaR, Historical Simulation and Monte Carlo Simulation, which are different methods but with common attributes and limits. Thus, regardless of the calculating method of VaR chosen by the banking manager function of the importance of the instrument and its specified qualities, VaR is currently the most frequent method used, being considered as a respectable banking risk measure.

Key words: method, option, portfolio, simulation.

The place and the level of banks in the economy are related to their intermediary quality in the savings-investments relationship, which is very important in the economic growth. Banks, being the intermediary between the depositors and the beneficiaries of savings, have the following activities: collecting funds, assuming debtors risks, that means analyzing the loans and assuming the associated risks, assuming the interest risk, because the intermediation suppose a maturity transformation.

From the accomplishment of these functions, banks gain recompense, representing the main source of the banking profit. The banking institutions, like other institutions producers of goods and services, have also a major purpose the responsibility to their shareholders, the maximization of the profits.

The bank management must have methods and techniques for reducing risk. In the banking sector, the risk must be considered as a interdependent risks complexity, being possible to have common causes or producing other risks.

Considering the market risk importance, its evaluation it is necessary to each bank applying the current measurement methods. In this context, I will present the measurement method Value at Risk (VaR) and calculating methods of VaR.

Value at Risk (VaR) is a measure of market risk which objectively combine the sensitivity of the portfolio to market changes and the probability of a given market change.

The VaR concept was developed during the 1970’s and the 1980’s, when some financial institutions have begun the study of internal models in measuring the risk as a whole. These institutions begun the study on models to serve their own risk management, but according their complexity, the study became more difficult, and most important, to be able to measure the risks considering their interdependence and the fact that their methodology does not cover this area. The most known system have been developed by JP Morgan, which have been operational during the 1990’s. This system based on the theory of the standard portfolio using estimations of standard deviations and correlations of the incomes of the different instruments, applied to the entirely institution by a measurement method of the global risk.

Although, VaR has some limitations that require the use of stress and scenario tests as a backup, overall, VaR is the best single risk measurement technique available.
The Basel Committee, trying to reduce the fragility of international active banks, demands banks to use the VaR method in establishing the minimum capital necessary at portfolios. As such, VaR has been adopted by the Basel Committee to set the standard for the minimum amount of capital to be held against market risks.

Value at Risk is a potential loss. Potential losses can theoretically extend to the value of the entire portfolio. In this way, VaR is the maximum loss in a certain period, at a preset confidence level. The confidence level is the probability that the loss exceeds this upper bound.

Value at Risk is defined as the value that can be expected to be lost during severe market fluctuations. Typically, a severe loss is defined as a loss that has 1% chance of occurring on any given day. If daily losses are measured, the equivalent of this is: "On average, we will lose VaR or more on two to three days per year”.

VaR can be defined as an expected loss to be exceeded with x% probability in a certain period of t days on the portfolio of credits measured.

When using the Value at Risk method, the manager of a portfolio of financial instruments is interested in making the following statement: "We are X percent certain that we will not lose more than V dollars in the next N days”, where V is VaR of the portfolio, N is the time horizon and X the confidence level.

A common assumption is that movements in the market have a Normal probability distribution, meaning there is 1% chance that losses will be greater than 2.32 standard deviations. Assuming a Normal distribution, 99% VaR can be defined as follows:

\[
VaR_T = 2.32\sigma_T
\]

where \(\sigma\) is the standard deviation of the portfolio’s value. This is illustrated in Figure 1.

T refers to the time period over which the standard deviation of returns is calculated. VaR can be calculated for any time horizon. For trading operations, a one day horizon is used. VaR for one day horizon is called DEaR: Daily Earnings at Risk (during the 1990’s).

For example, we consider an equity portfolio with a daily standard deviation of USD 10 million. Assuming a Normal distribution, the 99% confidence interval VaR is USD 23 million. We would expect that the losses would be greater than USD 23 million on 1% of trading days, or 2 to 3 days per year.

In practice, the VaR method can be difficult because it contains restrictive assumptions concerning the revenues of portfolios and their distribution in the portfolio.

It is necessary to understand that VaR is not the worst possible loss. Losses equal to the size of VaR are expected to happen several times per year; VaR is therefore not equal to capital.
Figure 1. The Relationship between VaR and Standard Deviation

As an alternative of the VaR measurement method is the conditional value at risk method CvaR, which is known as the average excessive loss, the average reduction or the VaR limit. CvaR is a coherent method of risk measurement having attractive properties, including convexity. Plus, the minimization of CvaR leads to a portfolio with a lower VaR.

Methods of calculating VaR

The most common methods of calculating VaR are: Parametric VaR, Historical Simulation and Monte Carlo Simulation. It is important to note that while the three calculation methods differ, they do share common attributes and limitations.

To make a comparison between the three methods, each method will be presented as follows.

**Parametric VaR**

Parametric VaR is a model known as Linear VaR, Variance-Covariance VaR, Greek-Normal VaR, Delta Normal VaR or Delta-Gamma Normal VaR. The method is parametric in that it assumes that the probability distribution is Normal and then requires calculation of the variance and covariance parameters. The method is linear in that changes in instrument values are assumed to be linear with respect to changes in risk factors. For example, for bonds the sensitivity is described by duration, and for options it is described by the Greeks.

The Parametric VaR method is as follows:

- Define the set of risk factors that will be sufficient to calculate the value of the bank’s portfolio;
- Find the sensitivity of each instrument in the portfolio to each risk factor;
- Obtain historical data on the risk factors to calculate the standard deviation of the changes and the correlations between them;
- Estimate the standard deviation of the value of the portfolio by multiplying the sensitivities by the standard deviations, taking to account all the correlations;
- Assume that the loss distribution is Normally distributed, and therefore, approximate the 99% VaR as 2.32 times the standard deviation of the value of portfolio.
The advantages of the Parametric VaR method:

- It is typically 100 to 1000 times faster to calculate Parametric VaR compared with Monte Carlo and Historical Simulation;
- The method allows the calculation of VaR contribution.

The limitations of the Parametric VaR method:

- It gives a poor description of nonlinear risks;
- It gives a poor description of extreme events, such as crises, because it assumes that the risk factors have a Normal distribution. In reality, the risk factor distributions have a high kurtosis (a measure of the peakness of a probability distribution of a random variable really evaluated) with more extreme events than would be predicted by the Normal distribution;
- The method uses a covariance matrix, and this implicitly assumes that the correlation between risk factors is stable and constant over time.

To give an intuitive understanding of Parametric VaR, we make some notes:

If we have a portfolio of two instruments, the loss on the portfolio \( L_p \) will be the sum of the losses on each instrument:

\[
L_p = L_1 + L_2
\]

The standard deviation of the loss on the portfolio \( \sigma_p \) will be:

\[
\sigma_p = \sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2
\]

Where \( \sigma_1 \) is the standard deviation of the losses from instrument 1 and \( \rho_{1,2} \) is the correlation between the losses from 1 and 2.

The Parametric VaR is suited to risk measurement problems where the distributions are known and reliably estimated. However, this condition is often not met in practice, especially when we have small sample sizes, and in such circumstances the method can be very unreliable.

**Historical Simulation VaR**

The Historical Simulation method is the simplest VaR technique, but it takes more time to run than Parametric VaR.

Historical Simulation is a nonparametric method of the risk measurement that estimates risk without making strong assumptions about the distribution. Instead of imposing some parametric distribution on the data, the nonparametric method let the data speak for themselves and estimates risk measures from the empirical distribution. Relative to parametric method, the nonparametric method has the major attraction that it avoids the danger of misspecifying the distribution, which could lead to major errors in estimated risk measures. It is based on the assumption that the near future will be sufficiently like the recent past that can be used the recent historical data to forecast the future. In practice, its usefulness depends on whether this assumption holds in any situation. Fortunately, it often does hold, and nonparametric method has a good track record. On the other hand, the method can be inaccurate where this assumption does not hold. Its estimates can be imprecise, especially in the tail regions where data are especially sparse. As a result, nonparametric method often has difficulty handling extremes.

Historical Simulation takes the market data for the last 250 days and calculates the percent change for each risk factor on each day. Each percentage change is then multiplied by today’s market values to present 250 scenarios for tomorrow’s values. For each of these scenarios, the portfolio is valued using full, nonlinear pricing models. The third worst day is then selected as being 99% VaR.

For example, let’s consider calculating the VaR for a five year zero-coupon bond paying USD 100. We start by looking back at the previous trading days and noting the five
year rate on each day. We then calculate the proportion by which the rate changed from one
day to the next:

\[ \Delta_t = \frac{r_{t+1} - r_t}{r_t} \]

Scenarios are then created for tomorrow’s rate by applying the proportional change to
today’s rate.

\[ r_{Scenario,k} = r_{Today} (1 + \Delta_t) \]

We shift from a subscript of \( t \) to \( k \) because there is a conceptual shift. We use data
from the past days to create scenarios of what could happen tomorrow. The scenarios
therefore do not represent what has happened, but what could happen in the next day. Using
these scenarios, we value the bond using the formula:

\[ Value_{Scenario,k} = \frac{100}{(1 + r_{Scenario,k})^5} \]

and then calculate the change in value:

\[ \Delta V_k = Value_{Scenario,k} - Value_{Today} \]

An example of Historical Simulation is illustrated in Table 1, with 10 days of data.
The Rate column shows the rate observed at the end of each day. The next columns show the
proportional change and the scenario that would occur if that change were to happen starting
from today’s rate. The final columns show the bond value and loss in each scenario. In this
example, the worst case change is the loss of 39 cents, which is a rough estimate of the 10
percentile loss.

<table>
<thead>
<tr>
<th>Data</th>
<th>Rate (%)</th>
<th>Proportional change (%)</th>
<th>Scenario Rate</th>
<th>Bond Value (USD)</th>
<th>Change in Value (USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09 July</td>
<td>5.02</td>
<td>0.6</td>
<td>5.03</td>
<td>78.25</td>
<td>-0.10</td>
</tr>
<tr>
<td>10 July</td>
<td>5.05</td>
<td>-0.9</td>
<td>4.96</td>
<td>78.52</td>
<td>0.17</td>
</tr>
<tr>
<td>11 July</td>
<td>5.01</td>
<td>1.9</td>
<td>5.09</td>
<td>78.00</td>
<td>-0.35</td>
</tr>
<tr>
<td>12 July</td>
<td>5.10</td>
<td>0.2</td>
<td>5.01</td>
<td>78.32</td>
<td>-0.03</td>
</tr>
<tr>
<td>13 July</td>
<td>5.11</td>
<td>-0.8</td>
<td>4.96</td>
<td>78.50</td>
<td>0.14</td>
</tr>
<tr>
<td>16 July</td>
<td>5.07</td>
<td>2.1</td>
<td>5.11</td>
<td>77.96</td>
<td>-0.39</td>
</tr>
<tr>
<td>17 July</td>
<td>5.18</td>
<td>1.0</td>
<td>5.05</td>
<td>78.17</td>
<td>-0.19</td>
</tr>
<tr>
<td>18 July</td>
<td>5.23</td>
<td>-0.2</td>
<td>4.99</td>
<td>78.40</td>
<td>0.05</td>
</tr>
<tr>
<td>19 July</td>
<td>5.22</td>
<td>-0.2</td>
<td>4.99</td>
<td>78.39</td>
<td>0.04</td>
</tr>
<tr>
<td>20 July</td>
<td>5.21</td>
<td>-4.0</td>
<td>4.80</td>
<td>79.10</td>
<td>0.75</td>
</tr>
<tr>
<td>23 July</td>
<td>5.00</td>
<td></td>
<td></td>
<td>78.35</td>
<td></td>
</tr>
</tbody>
</table>


The advantages of Historical Simulation:
- It is easy to communicate the results throughout the organization because the
  concepts are easily explained;
- There is no need to assume that the changes in the risk factors have a structured
  parametric probability distribution.

The disadvantages are due to using the historical data in such a raw form:
- The result is usually dominated by a single, recent, specific crisis and it is very
difficult to test other assumptions. The effect of this is that Historical VaR is
strongly backward-looking, meaning the bank is protecting itself from the last crisis, but not necessarily preparing itself for the next;

- There can be also an unpleasant *window effect*. When 250 days have passed since the crisis, the crisis observation drops out of our window for historical data and the reported VaR suddenly drops from one day to the next. This often causes traders to mistrust the VaR because they know there has been no significant change in the risk of the trading operation, and yet the quantification of the risk has changed dramatically.

**Monte Carlo Simulation VaR**

Monte Carlo Simulation is also known as Monte Carlo Evaluation (MCE). This method estimates VaR by randomly creating many scenarios for future rates, using nonlinear pricing models to estimate the change in value for each scenario, and then calculating VaR according to the worst losses.

The Monte Carlo Simulation method is a stochastic simulation ideally suited to a great range of risk measurement problems, and will often provide the best way of dealing with the problems we are likely to encounter. This is particularly good at dealing with complicating factors that other methods often cannot handle. Therefore, the stochastic method is a method of choice for the complex risk problems.

The advantages of Monte Carlo Simulation:

- Unlike Parametric VaR, Monte Carlo uses full pricing models and can therefore capture the effects of nonlinearities;
- Unlike Historical VaR, Monte Carlo can generate an infinite number of scenarios and therefore test many possible future outcomes.

The disadvantages of Monte Carlo Simulation:

- The calculation of Monte Carlo VaR can take 1000 times longer than Parametric VaR, because the potential price of the portfolio has to be calculated 1000 times;
- Unlike Historical VaR, Monte Carlo requires the assumption that the risk factors have a Normal or Log-Normal distribution.

The Monte Carlo Simulation method assumes that there is a known probability distribution for the risk factors. The usual implementation of Monte Carlo assumes a stable, Joint-Normal distribution for the risk factors.

This is the same assumption used for Parametric VaR. The method calculates the covariance matrix for the risk factors in the same way as Parametric VaR, but unlike Parametric VaR, it then decomposes the matrix. The decomposition ensures that the risk factors are correlated in each scenario. The scenarios start from today’s market condition and go one day forward to give possible values at the end of the day. Nonlinear pricing models are then used to value the portfolio under each of the end of day scenarios.

For bonds nonlinear pricing means using the bond pricing formula rather than duration, and for options, it means using a pricing formula such as Black-Scholes rather than just using the Greeks.

From the scenarios, VaR is selected to be the 1 percentile worst loss. For example, if 1000 scenarios were created, the 99% VaR would be the tenth worst result.

The Monte Carlo method is conceptually simple. The one mathematically difficult step is to decompose the covariance matrix in such a way as to allow creating random scenarios with the same correlation as the historical market data.

If we have just two factors, we can easily create correlated random numbers in the
following way:

- Draw a random number $z_1$ from a Standard Normal distribution;
- Multiply $z_1$ by the standard deviation of the first risk factor ($\sigma_A$) to create the first risk factor for that scenario, $f_A$:
  
  $$ f_A = \sigma_A z_1, \quad z_1 \sim N(0,1) $$

- Multiply $z_1$ by the correlation $\rho_{A,B}$;
- Draw a second independent random number $z_2$;
  - Multiply $z_2$ by the root of 1 minus the correlation squared $\left(\sqrt{1 - \rho_{A,B}^2}\right)$;
  - Add the two results together to create a random number ($y$) that has a standard deviation of one and correlation $\rho_{A,B}$ with $f_A$:
    
    $$ y = z_1 \rho_{A,B} + z_2 \sqrt{1 - \rho_{A,B}^2}, \quad z_2 \sim N(0,1) $$

- Multiply $y$ by the standard deviation of the second factor risk $\sigma_B$ to create the second risk factor for that scenario $f_B$:
  
  $$ f_B = \sigma_B y $$

This process can be summarized in the following equations:

$$ f_A = \sigma_A z_1, \quad z_1 \sim N(0,1) $$

$$ f_B = \sigma_B \left( z_1 \rho_{A,B} + z_2 \sqrt{1 - \rho_{A,B}^2} \right), \quad z_2 \sim N(0,1) $$

Unfortunately, this sample approach does not work if there are more than two risk factors. If there are many risk factors, we need to create the correlation by decomposing the covariance matrix using either **Cholesky decomposition** (after the French mathematician André-Louis Cholesky) or **Eigen-value decomposition** (concept introduced in 1924 by the mathematician Hilbert).

**Cholesky decomposition** finds a new matrix $A$, such the transpose of $A$ times $A$ equals the covariance matrix $C$. $A$ is required to be upper triangular, all the elements below the main diagonal are zero.

$$ C = A^T A $$

The algorithm used to find the Cholesky matrix does not work if the matrix is not positive definite. To be positive definite, all the Eigenvalues (see below) of the covariance matrix must be positive. In practical terms, this means that none of the risk factors can have a perfect correlation with another factor. In practice, Cholesky decomposition tends to fail if there are more than 10 to 20 risk factors in the covariance matrix.

The alternative to Cholesky decomposition is **Eigenvalue decomposition**, which is also known as Principal Components analysis. It is more difficult to program than Cholesky, but it will work for covariance matrices that are not positive definite. This means that it will work for covariance matrices with hundreds of risk factors. Eigenvalue decomposition only fails if different parts of the matrix were built with data from different time periods (because inconsistencies in the data may cause negative variances for some of the principal components). Eigenvalue decomposition has the advantage that it can give intuitive insights into the structure of the random risk factors, allowing identifying the main drivers of risk. This can help to reduce the number of simulations needed.

Eigenvalue decomposition works by looking for two matrices, $A$ and $E$, to satisfy the following equation:

$$ C = E^T A E $$

$C$ is the covariance matrix, $A$ is a square matrix in which all the elements other than the main
diagonal are zero, and $E$ is a matrix that multiplied by its transpose the result is the identity matrix.

Monte Carlo method takes long time to compute the results, compared with Parametric VaR. Creating the random scenarios for the risk factors takes relatively little time to run once it is programmed. The slow part of running Monte Carlo VaR is doing the pricing of all instruments under each scenario. Several techniques have been developed to reduce the computation time, where the most popular are:

- Parallel processing
- Stabilization of results
- Variance-reduction techniques
- Approximate pricing

*Parallel processing* uses several computers and simultaneously evaluates different groups of scenarios or instruments on each. Some care has to be taken to distribute the processes and collate the results, but this is relatively easy. The main drawback is the cost of hardware.

*Stabilization of results* reduces the number of scenarios that need to be run. If the scenarios are completely different on one day compared with the next, it is quite likely that the results will change not because there has been any fundamental change in the risk, but because different random scenarios have been tested. The common approach to reducing this problem is to run many scenarios each day to *average out* the random fluctuations.

An alternative approach is to run fewer scenarios but fix the Normally distributed, independent, random numbers that are used to create the scenarios and only allow the correlation matrix and portfolio composition to change from day to day. The consequence is that the results only change due to real market and portfolio changes.

The random numbers can be fixed either by generating the numbers once and storing them, or by using the same seed number to start the random sequence each day. Although fixing the random numbers sounds straightforward, it does require discipline when writing the program to ensure that the same random number will be used in the same place for every run. There is a slight disadvantage to fixing the random numbers because there is less opportunity to test obscure scenarios that may cause unusual losses. As a compromise, it may allow a proportion of the random numbers to change each day.

*Variance-reduction techniques* choose the numbers to be more evenly distributed than in the pure random case. This reduces the randomness in the results and allows using fewer scenarios. In some cases, the number of scenarios may be reduced by a factor of 10 to 100 while still maintaining the same accuracy.

In using Variance reduction, it is important to note that as the number of risk factors increases, it becomes more difficult to create well-balanced pseudorandom numbers. A common technique is to concentrate the random numbers on the most important risk factors and let the lesser factors be purely random.

*Approximate pricing* reduces the computation time for each instrument by simply taking less time to evaluate each scenario. This is done by using simple models that are not accurate, but can be run quickly. A typical example would be to use the Black-Scholes equation to approximate the value of an option that would be more accurately priced by binomial trees.

Unfortunately, no easy answer exists to the question of which of three methods presented is best, because the methods differ in function of the chosen instrument and its qualities. For example, for options (*investment risk*), the methods differ in ability to capture the risk of options and option-like instruments, ease of implementation, ease of explanation to...
managers, flexibility in analyzing the effect of changes in the assumptions and reliability of results. The best choice will be determined by which dimensions the risk manager considers most important. Table 2 presents a comparison table between the methods of calculating VaR for options, considering the qualities presented hereinbefore.

**Table 2. Comparison between the methods of calculating VaR for options**

<table>
<thead>
<tr>
<th>Quality</th>
<th>Historical Simulation</th>
<th>Parametric VaR</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is able to capture the risks of portfolios that include options?</td>
<td>Yes, regardless of the option content of the portfolio</td>
<td>No, except when computed using a short holding period for portfolios with limited or moderate option content</td>
<td>Yes, regardless of the option content of the portfolio</td>
</tr>
<tr>
<td>It is easy to implement?</td>
<td>Yes for portfolios for which data on the past values of the market factors are available</td>
<td>Yes for portfolios restricted to instruments and currencies covered by available off-the-shelf software; otherwise, reasonably easy to moderately difficult to implement, depending on the complexity of the instruments and availability of data</td>
<td>Yes for portfolios restricted to instruments and currencies covered by available off-the-shelf software; otherwise, moderately to extremely difficult to implement</td>
</tr>
<tr>
<td>Are the computations performed quickly?</td>
<td>Yes</td>
<td>Yes</td>
<td>No, except for relatively small portfolios</td>
</tr>
<tr>
<td>It is easy to explain to managers?</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Does it produce misleading VaR estimates when recent past is atypical?</td>
<td>Yes</td>
<td>Yes, except that alternative correlations/standard deviations may be used</td>
<td>Yes, except that alternative estimates of parameters may be used</td>
</tr>
<tr>
<td>Are „what-if” analyses to examine effect of alternative assumptions easy to perform?</td>
<td>No</td>
<td>Examining alternative assumptions about correlations/standard deviations is easy; examining alternative assumptions about the distribution of the market factors is impossible</td>
<td>Yes</td>
</tr>
</tbody>
</table>


In conclusion, VaR is a summary statistical method of measuring normal market risk. VaR summarizes the information in the probability distribution of possible changes in portfolio value in a particular way. Moreover, VaR is based on a range of assumptions, few of which will be satisfied exactly. Although, VaR method does not contain all the information about market risks, this method often is being used together with other methods such as the Greeks, Stress Testing and Scenario Testing. In retrospect, it is clear that VaR was much overrated and is now discredited as a respectable banking risk measure.
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