LINEAR PROGRAMMING MODELS APPLIED IN SELECTION OF PARTNERS IN A VIRTUAL ORGANISATION

Radu Serban, Phd
Iulian Mircea, Phd
Student Daniela Todose, Phd
The Bucharest Academy of Economic Studies

Abstract:

In this paper, we consider how the selection of partners in a virtual organization (VO) can be assisted through mixed integer linear programming (MILP) models, when the configuration of VO is based on a virtual organization breeding environment (VBE). Apart from basic model, which focuses on the minimization of fixed and variable costs, we present extension that accommodate transportation costs.

Keywords: partner selection, virtual organization, optimization, multiple criteria, mixed integer linear programming.

1 Introduction

One definition of a virtual organization is that it is a temporary network of companies that comes together quickly to exploit fast-changing opportunities. The new organizational structure, the virtual (V-form) organizational structure, is emerging. In many cases it is replacing the multidivisional (M-form) structure (this organizational innovation involved substituting quasi-autonomous operating divisions), just as the M-form structure replaced the unitary form (U-form) structure (the principal operating units in the U-form firm are the functional divisions—sales, finance, manufacturing, etc.), because of the need for firms to remain competitive given environmental changes. Several factors are driving businesses toward the use of the virtual organizational structure.

First, the pace of business is continually increasing with shorter product life cycles requiring quicker response to market opportunities. Second, the cost of market entry is often smaller than previously, especially in the information services and other technology-driven industries. Third, corporations are now driven more by customer demands than by internal needs. And finally, there is an increased need for globalization to remain competitive. Collaborative networks are becoming more important in global and regional business, thanks to their ability to combine organizational competences. But as individual companies seek efficiency gains by focusing on their core competences while outsourcing non-core operations, the degree of inter-firm transactions grows considerably. This makes it imperative to manage network relations well, which in turn calls for the development and deployment of decision support models that assist companies in the management of these relations. In this paper, we focus on the problem of selecting VO partners in a VBE. That is, when a VBE identifies a business opportunity, it has to determine a 'good' VO configuration for meeting the identified customer need; this is essentially an optimization problem that can be formulated as a mixed integer linear programming (MILP) model. To support the VBE in solving it, we develop a model for allocating work among potential VO partners, taking into account fixed and variable work costs, transportation costs, as well as inter-organizational dependencies.

2. Mathematical Methods for Virtual Organization Partner Selection

VO partner selection can be treated as a work-allocation problem.

Let $M=\{1,2,\ldots,m\}$ denote the set of candidate partners in the VBE. At the outset, the VBE identifies a business opportunity which is to be addressed by carrying out a project for the Customer. The project tasks are denoted by $N=\{1,2,\ldots,n\}$, and task $j\in N$ involves a workload w_j which is measured in relevant units (e.g. person months). We note:

$C_{i,j}$ distribution for capacity of candidate *i* on task j

$c_{i,j}^{k}$ k-th element of $C_{i,j}$

$e_{a,b}$ intensity of earlier collaboration between candidates a and b

g_i fixed cost of candidate *i* work on the project

$g_{i,j}$ fixed cost of candidate *i* work on task j of the project

m number of candidates

n number of tasks in the project

$p_{i,j}(k)$ probability that candidate *i* realised capacity on task *j* is $C_{i,j}$

$t_{a,b}$ unit transportation cost between candidates *a* and *b*

$v_{i,j}$ variable cost of candidate *i* work on task *j*

w_j workload of task j

$\delta_{r',r''}$ quantity of transportation required between tasks *r'* and *r"*

$\rho_{i,j}^{RISK}$ capacity risk of *i* work on task *j*, using risk measure RISK

Variables: $x_{i,j}$ candidate *i* work allocation on task *j*

y_i takes value one if *i* is selected into the VO, zero otherwise

$y_{i,j}$ takes value one if *i* performs work on task *j*, zero otherwise

$z_{a,b}$ takes value one if both candidates *a* and *b* are selected into the VO, zero otherwise

$z_{a,b}^{r}$ takes value one if candidates *a* and *b* perform tasks *r'* and *r"*, respectively, and transportation is required between tasks *r'* and *r"*; zero otherwise

Let $y_i=\begin{cases}0 & \text{if } x_{i,j}=0 \text{ for } \forall\, j\in N\\ 1 & \text{if } x_{i,j}>0 \text{ for at least one } j\in N\end{cases}$, $y_{i,j}=\begin{cases}0 & \text{if } x_{i,j}=0\\ 1 & \text{if } x_{i,j}>0\end{cases}$

Thus, y_i is equal to one if and only if candidate *i* performs some work in the project, and zero otherwise, and $y_{i,j}$ indicates whether or not some work on task *j* is allocated to candidate *i*. Without loss of generality, it can be assumed that $c_{i,j}^{k}$ are sorted in descending order so that $c_{i,j}^{1}=\max_k c_{i,j}^{k}$.

The decision variable is the work-allocation matrix $X_{m\times n}$ having the elements $x_{i,j}$. Our basic model has a single cost criterion which accounts for the candidates' variable and fixed costs, i.e.,

$$\min_{X,Y} COST(X,Y)=\sum_{i=1}^{m} g_i\cdot y_i+\sum_{j=1}^{n}\sum_{i=1}^{m}\left(g_{i,j}\cdot y_{i,j}+v_{i,j}\cdot x_{i,j}\right) \qquad (1)$$

where the matrix $Y=\left(y_{i,j}\right)$.

In the objective function, the first term (I: $\sum_{i=1}^{m} g_i \cdot y_i$) is the sum of fixed costs due to the addition of partners to the VO, while the second term (II: $\sum_{j=1}^{n}\sum_{i=1}^{m}\left(g_{i,j} \cdot y_{i,j} + v_{i,j} \cdot x_{i,j}\right)$) covers the fixed and variable costs due to the work that the partners perform on their respective tasks. This function is flexible in that some costs can be ignored if they are irrelevant.

The two types of constraints in the optimization problem ensure that all project demands are met, and that the optimal solution is feasible. Starting with project constraints, the workload of each task has to be completed: $\sum_{i=1}^{m} x_{i,j} \geq w_j, \quad \forall\, j \in N$ (2)

Furthermore, the workload that is allocated to a candidate must not exceed its maximum capacity:

$$0 \leq x_{i,j} \leq c_{i,j}^{k}, \quad \forall\, i \in M,\ j \in N, \qquad (3)$$

Continuing with feasibility constraints, correct values for binary y_i -s are ensured by the constraints:

$$\frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon \leq y_i \leq \frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon + 1, \quad \forall\, i \in M \qquad (4)$$

Here, the numerators denote the total amount of work that is allocated to candidate i while the denominator is the total workload of the project: thus, these quotients are equal the proportion of the projects' workload that is allocated to candidate *i*. Furthermore, ε corresponds to the proportion of the total workload that a candidate has to exceed in order to be considered a relevant VO partner. In consequence, $y_i = 1$ if at least $\varepsilon \times 100$ percent of the projects' workload is allocated to candidate *i*, and $y_i = 0$ otherwise.

In the first expression of (3), the denominator is needed to keep the right hand side below one. Otherwise, if the right hand side increased above one, the model would become infeasible because y_i is a zero-one variable. Similarly, in the latter expression, the denominator is needed to push the right hand side below one if not enough work is allocated to candidate *i*. The following constraints ensure that the binary y_{ij} assume correct values:

$$y_{ij} \geq \frac{x_{ij}}{c_{ij}^{1}}, \quad \forall\, i \in M,\ j \in N \qquad (5)$$

That is, $y_{ij} = 1$ if at least some work of task *j* is allocated to candidate *i*, and $y_{ij} = 0$ otherwise. No upper constraint for y_{ij} is needed, because increasing these binary variables from zero to one results in higher total costs, meaning that the y_{ij} remain at zero level if this is feasible. If one introduces additional decision criteria such that the benefit increases when $y_{ij} = 1$, then an upper bound similar to that for y_i becomes necessary.

In summary, basic optimization model can now be stated as

$$\min_{X,Y} COST(X,Y) = \sum_{i=1}^{m} g_i \cdot y_i + \sum_{j=1}^{n}\sum_{i=1}^{m}\left(g_{i,j} \cdot y_{i,j} + v_{i,j} \cdot x_{i,j}\right)$$

$$\sum_{i=1}^{m} x_{i,j} \geq w_j, \quad \forall\, j \in N,$$

$$0 \leq x_{i,j} \leq c_{i,j}^{k}, \quad \forall\, i \in M,\ j \in N$$

$$\frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon \leq y_i \leq \frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon + 1, \quad \forall\, i \in M$$

$$y_{ij} \geq \frac{x_{ij}}{c_{ij}^{1}}, \quad \forall\, i \in M,\ j \in N$$

$$y_i \in \{0,1\},\ y_{ij} \in \{0,1\},\ \forall\, i \in M,\ j \in N.$$

Transportation costs are caused mainly by two factors, 1) geographical distance and 2) volume and weight of the cargo. For instance, it may be possible to compensate the cheaper labor costs of a far-away manufacturing site by the lower transportation costs from a near-by site. Thus, the operational costs discussed anterior must be augmented by considering transportation costs, too. This can be achieved only by explicating the sequence of tasks that are involved in the assembly of the physical product. For instance, consider a project of three tasks. Assume that output of Task 1 must be made available to the same site where Task 3 is carried out and that the volume of this transportation is 5 units. The corresponding task sequence can be illustrated with the simple network. Because Task 2 does not have physical connection to Tasks 1 or 3, it is shown as a disconnected node. For instance, Task 1 could correspond to the manufacturing of a microchip, which is assembled into the end-product in Task 3. Task 2, in turn, could represent software development for the end-product.

Let $r = (r', r'')$ denote a pair of tasks such that the output of task r' must be at the same location where task r'' is carried out. Let R denote the set of all such pairs. For each $(r', r'') \in R$, let $\delta_{r',r''}$ be the corresponding output volume of task r'. The unit cost of transportation can be presented as a graph whose nodes correspond to the candidate partners and whose edges represent the unit transportation costs between adjacent nodes; specifically, for candidates a and b, these unit costs are denoted by $t_{a,b}$. For each pair $(r', r'') \in R$, we have two sets of candidates, i.e., (i) those that are capable of performing task r' and (ii) those that are capable of performing task r''. These two sets are connected by edges between the candidates, such that each edge represents the transportation cost from one candidate to another, in accordance with the relation (r', r''). Transportation costs can now be incorporated into our MILP model as follows. For any given pair of tasks $r = (r', r'')$, we define the binary variable $s_{a,b}^{r}$: $s_{a,b}^{r} = \begin{cases} 0, & \text{if } y_{a,r'} = 0 \text{ or } y_{b,r''} = 0 \\ 1, & \text{if } y_{a,r'} = 1 \text{ and } y_{b,r''} = 1 \end{cases}, \quad \forall\, r \in R, a, b \in M$, $c_{a,r'}^{1} \geq w_{r'}$ and $c_{a,r''}^{1} \geq w_{r''}$, where this definition applies for all pairs of candidates a, b such that a is capable of performing task r' and b can perform task r''. Thus, $s_{a,b}^{r}$ is one if tasks r' and r'' are enabled by transportation between candidates a and b; otherwise $s_{a,b}^{r}$ is zero. In addition, the following constraints are needed: $s_{a,b}^{r} \leq \frac{y_{a,r'} + y_{b,r''}}{2}$ and $s_{a,b}^{r} \geq y_{a,r'} + y_{b,r''} - 1$

(6)

The first of these constraints ensures that $s_{a,b}^{r}$ is zero if tasks r' and r'' are not allocated to candidates *a* and *b,* respectively. The second constraint ensures that $s_{a,b}^{r}$ is one if candidates a and b work on tasks r' and r'', respectively. The total transportation costs can now be written as $CTR = \sum_{r \in R} \delta_{r',r''} \cdot t_{a,b} \cdot s_{a,b}^{r}$.

The above cost function is linear, thus the objective function (1) remains linear even when transportation costs are accounted for. In this case the optimization model is

$$\min_{X,Y} CT(X,Y) = \sum_{i=1}^{m} g_i \cdot y_i + \sum_{j=1}^{n} \sum_{i=1}^{m} \left(g_{i,j} \cdot y_{i,j} + v_{i,j} \cdot x_{i,j} \right) + \sum_{r \in R} \delta_{r',r''} \cdot t_{a,b} \cdot s_{a,b}^{r} \text{ with constraints:}$$

$$\sum_{i=1}^{m} x_{i,j} \geq w_j, \quad \forall\, j \in N,$$

$$0 \leq x_{i,j} \leq c_{i,j}^{k}, \quad \forall\, i \in M,\ j \in N,$$

$$\frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon \leq y_i \leq \frac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} w_j} - \varepsilon + 1, \quad \forall\, i \in M,$$

$$y_{ij} \geq \frac{x_{ij}}{c_{ij}^{1}}, \quad \forall\, i \in M,\ j \in N,\ y_i \in \{0,1\},\ y_{ij} \in \{0,1\}, \forall\, i \in M,\ j \in N,$$

$$s_{a,b}^{r} \leq \frac{y_{a,r'} + y_{b,r''}}{2},\ s_{a,b}^{r} \geq y_{a,r'} + y_{b,r''} - 1.$$

Under normal conditions, we can assume that infrastructure is reliable and therefore a transportation partner is available. However, if selecting the right transportation partner is crucial to the success of the project, then each $r \in R$ can be associated with a new task of the project. Thus, the selection of partners for these tasks is done similarly to other tasks.

In this way, the decision maker (DM) can also cater for possible risks related to transportation. The collaborating entities can be individual workers, intra-organizational teams, business units, or distinct companies, for instance. The level at which collaboration is analyzed depends on the case at hand. Nevertheless, work performed in collaboration causes transaction costs that would not exist if one entity performed the job. At the partner selection phase of VO creation it is unrealistic to estimate the transaction costs that arise during the VO life-cycle. Therefore, it is more practical to study non-monetary indicators that influence the size of transaction costs over the VO life-cycle. One such indicator – which can be measured relatively easily – is the number of past collaboration activities between partner candidates. It is reasonable to assume that the more the companies have collaborated earlier, the better they know each other's ways of action, which reduces the transaction costs of collaboration. In contrast, examples of measurable indicators that may increase transaction costs are geographical distance and linguistic difference. When used as partner selection criteria in VO configuration, we refer to these indicators as network preparedness criteria. The network preparedness criteria differ from traditional selection criteria in that their measurement involves two or more companies (i.e., one cannot measure 'geographical distance' for a single company). The traditional selection criteria are usually applicable to a single company. The measurement of inter-organizational performance is more viable in the management of a VBE than in an "open universe" of organizations. This is because the VBE members collaborate repeatedly, which permits the collection of data about inter-organizational performance. Thus, considerations such as trust, success of past collaboration, and congruence between

organizational culture and objectives can be employed as potentially useful criteria for VO partner selection in a VBE. For the calculations exists the lp_solve software (http://groups.yahoo.com/group/lp_solve/).

The MILP model can be extended through modifications from problems such as capital budgeting, job-shop scheduling, and portfolio selection, which all have connections to partner selection model.

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